Modeling and algorithm of attitude measurement of moving target by laser tracking systems

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Abstract. We demonstrate theoretically the feasibility of the full-attitude measurement of a moving target by laser tracking systems. Based on the combination of retroreflector target, laser tracking, and interferometry techniques, two kinds of laser tracking systems known as distance-angle-measurement (DAM) and distance-only-measurement (DOM) systems are proposed for the attitude measurement of a moving target in real time. Mathematical models and algorithms of both systems are built to determine and calculate the dynamic attitude of the target under the world coordinate system and the target's initial coordinate system respectively. The coordinate transform between different coordinate systems and the initial geometric parameters of the laser tracking stations can be self-calibrated. The coordinate transform self-calibration algorithm can be solved by the singular matrix decomposition method, while the self-calibration algorithm of the initial geometric parameters of the DOM system can be solved by the Levenberg-Marquardt method. The simulation results show that the self-calibration algorithm converges rapidly and its accuracy is satisfactory under noise measurement. © 2003 Society of Photo-Optical Instrumentation Engineers. [DOI: 10.1117/1.1534595]

Subject terms: laser tracking; attitude measurement; self-calibration; distance-angle measurement; distance-only measurement.

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1 Introduction

The measurement of position and attitude under dynamic conditions is required in modern metrology for industrial applications. In some situations, the attitude of a moving target is more important and more informative than only its coordinates. For example, the attitude or orientation or azimuth is more important data in the process of assembling parts and in determining the spatial path to moving or flying objects, such as the attitude of the end-effector of a robot hand. To meet these demands, laser tracking measurement technology has been developed in the past decade.1-13

Laser tracking systems can be classified into two types based on the number of tracking stations and the principles followed to calculate the target coordinates. The first type, using only a single tracking station, was proposed by Lau et al.1-3 and we define this kind of system as a distance-angle-measurement (DAM) system since both distance and angle are measured to acquire the target coordinates, which is like the electronic theodolite or the theodolite total working station system. In the DAM system, the target coordinates are calculated by the spherical coordinate method, where the relative distance from the tracking mirror to the target is measured with a laser interferometer and the rotation angles of the tracking mirror are measured with two encoders. In the theodolite system, however, the triangulation method is exploited to calculate the target coordinates, and the target angle information is also measured by encoders, but the distance between the measurement stations is precalibrated using a standard tool bar.14 Another type of laser tracking system using multiple stations was put forward by Nakamura et al.4,5 and Nakamura and Goto5,7 and we define it as distance-only-measurement (DOM) system since only the distance parameter is measured to calculate the target coordinates. In the DOM system, the multilateration technique is adopted to measure the target coordinates.

To measure the attitude of the target, Lau and Hocken2 made some modifications to their measurement system and realized partial attitude measurement, five degrees of freedom (DOF). Inspired by Lau and Hocken’s work, Prenninger et al.8 proposed a full attitude measurement system including a cube corner and a CCD camera to measure roll, pitch, and yaw. However, some intrinsic limits of measurement range and accuracy exist in his system because of the usage of a cube corner and a CCD camera. Hughes et al.13
tried to design a high-accuracy coordinate measurement machine (CMM) based on multilateration techniques using laser tracking stations, and their final design will probably use eight measurement stations and four targets to achieve a six DOF measurement capability. But they reported only a pseudo-multi-lateration experiment result using one tracking station; also neither algorithms nor setups are provided in their reference to show how to measure the six DOF of the object using eight stations and four targets.

In this paper, for the first time we systematically put forward how to measure the attitude of the moving target under different coordinate systems by the DAM and DOM laser tracking systems respectively. The proposed DAM system comprises three stations and three targets, while the DOM system requires at least six stations and three targets. Section 2 presents the principle and modeling to measure the full attitude of the target by DAM and DOM systems. Section 3 shows the details of algorithms and simulation results for the attitude measurement. Section 4 presents our summary and discussion.

2 Principle and Modeling

2.1 DAM System

Figure 1 shows the system consisting of three DAM laser tracking stations and a combination retroreflector target system, which cannot only track and measure the spatial coordinates of any retroreflector under dynamic conditions but also can measure the attitude of the target. Figure 2 shows the typical configuration of each station, which can track and measure the coordinates of the corresponding subtarget, such as retroreflector A. Part of the beam from the laser interferometer, which passes through a beamsplitter (BS), is directed to retroreflector A by the tracking mirror. When the beam returns in parallel from retroreflector A, it is divided into two beams. One beam goes back to the interferometer to measure the optical path variation, while the other one is reflected by the BS onto a 2-D position sensitive detector (PSD). When retroreflector A moves in space, the position of the laser beam on the PSD will change twice in the corresponding direction, and the output of the PSD will not be zero. The control unit will drive the tracking mirror to rotate according to the output of the PSD and cause the output of the PSD to move toward zero. The rotation of the tracking mirror is measured by two coaxially installed encoders. When the initial optical path between the center of the tracking mirror and retroreflector A is calibrated, the spherical coordinates of retroreflector A can be calculated in real time from the outputs of the encoders and the interferometer.

To measure the full attitude of the moving target, it is necessary to design a combination target system with three or more retroreflectors. The proposed combination target system consists of three retroreflectors, as shown in Fig. 1. The three retroreflectors A, B, and C are rigidly fixed on a frame and there is no rotation or translation between them. To have a wide view field by the ground stations, "cat's eye" retroreflectors with the refractive index \( n = 2 \) are suggested to be the target retroreflectors as they have a working range close to 180 deg. We built the Cartesian target coordinate system (TS) on the combination retroreflector system by setting A on the origin \( O' \), B on the positive direction of the \( x \) axis, \( C \) in the \( x'y'o'y' \) plane. The relative position relationship of retroreflectors A, B, and C can be derived from the mechanical design data or from the post-calibration procedure.

All three stations in Fig. 1 have tracking and measurement functions, as described in Fig. 2, and the three tracking stations (station 0, 1, and 2) aim and track the three retroreflectors A, B, and C on the target system, respectively. The initial lock of the retroreflectors can be accomplished by locating all of the tracking stations relative to a known reference coordinate system and this lock can be maintained by the control unit to move the outputs of the PSD toward zero. When the coordinates of every retroreflector on the target object are measured by the preceding multistation measurement system, the full attitude of the target system, that is, coordinates \( x, y, \) and \( z \) and roll, pitch, and yaw under the Cartesian system, can be determined.

To calculate the six DOF of the target system, we introduce two other kinds of coordinate systems, namely, the world coordinate system (WS) and the relative coordinate system (RS), as shown in Fig. 1. The WS is built on tracking station 0 and the coordinate under the WS is called as absolute coordinate. The RS is built on tracking stations 1 and 2, called RS 1 and RS 2, respectively. For the WS, the origin \( O \) is located at the rotation center on the tracking mirror, the \( y \) axis is along the initial direction of the horizontal rotating axis of the tracking mirror, and the \( z \) axis is along the vertical rotating axis of the tracking mirror. The whole system complies with the right-hand principle. RS 1 and RS 2 are built in the same way. To measure the six DOF of the target, two kinds of unknowns should be deter-

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Fig. 1 Schematic diagram of a total attitude measurement system for a moving combination target based on the DAM system.

Fig. 2 Schematic diagram of every tracking station.
Table 1: Original coordinates of points A, B, and C under different coordinate systems

<table>
<thead>
<tr>
<th></th>
<th>Measuring Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>WS</td>
<td>(x_{10}, y_{10}, z_{10})</td>
</tr>
<tr>
<td>RS1</td>
<td>(x_{10}, y_{10}, z_{10})</td>
</tr>
<tr>
<td>RS2</td>
<td>(x_{10}, y_{10}, z_{10})</td>
</tr>
<tr>
<td>TS</td>
<td>(0,0,0)</td>
</tr>
</tbody>
</table>

In this paper, the authors describe a moving combination target based on the DOM system. They discuss the schematic diagram of a total attitude measurement system and provide details on the coordinate systems and measurements involved. The system involves tracking stations that measure the coordinates of subtargets to determine the six angles of the tracking mirrors. The relative distances and attitudes of the moving target are measured in real-time by stations 0, 1, and 2, which are employed in the DOM system. The coordinates of the stations, together with the initial distances from the stations to the subtargets, must be known in advance, so that the coordinates of the subtargets can be deduced. These two kinds of parameters can be self-calibrated, which is also described in detail in Sec. 3. In addition, the attitude of the target system can be calculated under the initial coordinate system (IS), which is defined and shown in Sec. 3.1.5.

2.2 DOM System

Figure 3 shows another scheme using six tracking stations and a combination target system to measure the full attitude of the moving target based on the DOM system. In this proposed system, it is not necessary to measure the rotation angles of the tracking mirrors, and only the relative displacement of the target is measured to determine the six DOF of the object. Thus, the two coaxial encoders in each tracking station are not employed in the DOM system, and the DOF of the object. Thus, the two coaxial encoders in each tracking station are not employed in the DOM system, and only the relative displacement of the moving target based on the DOM system. In this proposed system, the initial distance from one station to the corresponding subtarget can be determined by using the station to track the subtarget while using another station to measure the same subtarget’s displacement along the incident light beam, as shown in Fig. 5. Subtarget P moves from point P_2 to point P_1, which is along the incident light beam of another tracking station. The distance D is also measured by this tracking station. The to-be-calibrated station measures the corresponding rotation angles of the two

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orthogonal axes of the tracking mirror—\(\alpha_1\), \(\alpha_2\), \(\beta_1\), and \(\beta_2\)—and the changes of incident beam path—\(\Delta l_1\) and \(\Delta l_2\). Then the following equation can be achieved to acquire the initial distance \(l_0\):

\[
D^2 = (l_0 + \Delta l_1)^2 + (l_0 + \Delta l_2)^2 - 2(l_0 + \Delta l_1)(l_0 + \Delta l_2)K, \tag{3}
\]

where

\[
K = \sin \beta_1 \sin \beta_2 \cos(\alpha_1 - \alpha_2) + \cos \beta_1 \cos \beta_2. \tag{4}
\]

### 3.1.2 Self-calibration of the coordinate transform between the WS and the RS

Under different coordinate systems, the coordinates of the target are different, and thus it is necessary to determine the transform matrices between different systems. Usually, homogeneous coordinate transform is employed. The rotation matrix and translation vectors between the WS and the RS of laser tracking system can be self-calibrated, as shown in Fig. 6. For simplicity, only RS 1 is discussed.

To determine the rotation matrix \(\mathbf{R}\) and translation vector \(\mathbf{T}\) between the WS and RS 1, the laser tracking stations 0 and 1 are set to track the same subtarget \(P\) and measure its coordinates simultaneously. The coordinates of \(P\) are \(\mathbf{d}_i\) under the WS and are \(\mathbf{m}_i\) under RS 1, where \(i = 1, 2, \ldots, N\) (\(N\) is the sampling number). Then the relation between \(\mathbf{m}_i\) and \(\mathbf{d}_i\) can be expressed by the following equation:

\[
\mathbf{d}_i = \begin{bmatrix} \mathbf{R} & \mathbf{T} \end{bmatrix} \mathbf{m}_i + \mathbf{v}_i \tag{5}
\]

where

\[
\mathbf{d}_i = \begin{bmatrix} x_{P_i} \\ y_{P_i} \\ z_{P_i} \end{bmatrix} \text{ and } \mathbf{m}_i = \begin{bmatrix} x_{1P_i} \\ y_{1P_i} \\ z_{1P_i} \end{bmatrix}
\]

are the coordinates of subtarget \(P\) under the WS and RS 1; \(\mathbf{v}_i\) is a \(3 \times 1\) noise vector; and \(\mathbf{T}\) is \(3 \times 1\) translation vector,

\[
\mathbf{T} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \tag{6}
\]

where \(a, b,\) and \(c\) are the coordinates of the origin of RS 1 under the WS. Here \(\mathbf{R}\) is the \(3 \times 3\) rotation matrix:

\[
\mathbf{R} = \begin{bmatrix} \cos \gamma \cos \beta & \cos \gamma \sin \beta \sin \alpha - \sin \gamma \cos \alpha & \cos \gamma \sin \beta \cos \alpha + \sin \gamma \sin \alpha \\ \sin \gamma \cos \beta & \sin \gamma \sin \beta \sin \alpha + \cos \gamma \cos \alpha & \sin \gamma \sin \beta \cos \alpha - \cos \gamma \sin \alpha \\ -\sin \beta & \cos \sin \alpha & \cos \cos \alpha \end{bmatrix}
\tag{7}
\]

where \(\alpha, \beta,\) and \(\gamma\) are the rotation angles around the \(x, y,\) and \(z\) axes of the WS, respectively. Note that each column vector of \(\mathbf{R}\) is a representation of the axis unit vector of the RS expressed in terms of the axis unit vector of the WS.

Measurement errors are inevitable during the measurement, thus the following least-squares equations are used to determine the values of \(\mathbf{R}\) and \(\mathbf{T}\):

\[
e^2 = \sum_{i=1}^{N} \| \mathbf{d}_i - \hat{\mathbf{R}} \mathbf{m}_i - \hat{\mathbf{T}} \|^2, \tag{8}
\]

where \(\hat{\mathbf{R}}\) and \(\hat{\mathbf{T}}\) are the optimal solutions of \(\mathbf{R}\) and \(\mathbf{T}\); \(e\) is the residuary; and \(\| \cdot \|\) expresses the norm. There are many methods to solve the Eq. (8). In this paper, we will present how to solve the equation using the singular matrix decomposition method (SMDM).

The partial derivative of \(e^2\) with respect to \(\mathbf{T}\) is as following:

\[
\frac{\partial e^2}{\partial \mathbf{T}} = \sum_{i=1}^{N} 2(\mathbf{d}_i - \hat{\mathbf{R}} \mathbf{m}_i - \hat{\mathbf{T}})(-1). \tag{9}
\]

When \(e^2\) is minimum, the preceding equation is equal to zero:
\[
\sum_{i=1}^{N} (d_i - \hat{R}m_i - \hat{T}) = 0.
\]  

Then
\[
\hat{T} = \hat{d} - \hat{R}m.
\]

where
\[
\hat{d} = \frac{1}{N} \sum_{i=1}^{N} d_i, \quad \hat{m} = \frac{1}{N} \sum_{i=1}^{N} m_i.
\]

The variables of \( \hat{R} \) and \( \hat{T} \) in Eq. (10) can be determined by moving the two origins of the WS and RS 1 to one point, so that \( \hat{T} \) can be disregarded first. After determining \( \hat{R} \), \( \hat{T} \) can be deduced from Eq. (11). As \( m \) and \( d \) represent the coordinates of the same point \( P \) under different coordinate systems, two point aggregates \( \{m_i\} \) and \( \{d_i\} \) have the same geometric center. Therefore \( \varepsilon^2 \) will be minimized when the origins of the WS and RS 1 coincide with the center of \( \{m_i\} \) and \( \{d_i\} \).

To calculate \( \hat{R} \), let
\[
d_{xi} = d_i - \bar{d}, \quad m_{xi} = m_i - \bar{m}.
\]

Then, because of Eq. (11), Eq. (8) will be
\[
\varepsilon^2 = \sum_{i=1}^{N} \|d_{xi} - \hat{R}m_{xi}\|^2 = \sum_{i=1}^{N} (d_{xi}^T d_{xi} + m_{xi}^T m_{xi} - 2d_{xi}^T \hat{R}m_{xi}),
\]

where \((\cdot)^T\) expresses the transpose. In the preceding formula, when the last item is maximal, then \( \varepsilon^2 \) will be minimal. Define matrix \( \mathbf{H} \) as
\[
\mathbf{H} = \sum_{i=1}^{N} m_{xi} d_{xi}^T.
\]

If \( \mathbf{H} \) can be singularly decomposed as:
\[
\mathbf{H} = \mathbf{UAV}^T
\]

where \( \mathbf{U} \) and \( \mathbf{V} \) are both the orthogonal matrices, \( \mathbf{A} \) is the diagonal matrix. The optimal solution of rotation matrix \( \hat{R} \) can be obtained by
\[
\hat{R} = \mathbf{VU}^T.
\]

After \( \hat{R} \) is solved, \( \hat{T} \) can be derived from Eq. (11).

In this case, it is supposed that \( \mathbf{H} \) can be singularly decomposed, which requires the subtarget’s trajectory is not in the same plane and the determinate of \( \mathbf{R} \) should be positive. If the requirements cannot be satisfied, other method must be used to solve Eq. (8), such as the quaternary unit method.\(^{16}\)

### 3.1.3 Simulation

To verify the effectiveness of the preceding self-calibration algorithm, we present the simulation results in this section. The mathematical models of the coordinate transform between the WS and the RS are described in Sec. 3.1.2.

In Eq. (8), let the nominal values of \( \mathbf{T} \) be \( (a,b,c)^T = (0,0,500)^T \); and let \( \alpha, \beta, \gamma = (45 \text{ deg}, 45 \text{ deg}, 60 \text{ deg}) \). Suppose the subtarget \( P \) moving along a spatial line \( P_1 \rightarrow P_2 \rightarrow P_3 \rightarrow P_4 \), as shown in Fig. 6. The coordinates of \( P_1, P_2, P_3, \) and \( P_4 \) are \((100, 100, 0), (300, 300, 0), (300, 300, 300), \) and \((300, 500, 300)\), respectively. We assume the unit of the coordinates in simulation is millimeters. In simulation, let the sampling number in every line \( P_1P_2, P_2P_3, \) and \( P_3P_4 \) be 10. As the lines \( P_1P_2, P_2P_3, \) and \( P_3P_4 \) are not in the same plane, the SMDM is used to determine the \( \mathbf{R} \) and \( \mathbf{T} \) between the WS and RS 1.

Considering the actual measuring accuracy of laser interferometer is less than 1 \( \mu \text{m} \) at the range of 1 m, we assume that there is a random measurement error 0 to 1 \( \mu \text{m} \) in the coordinates of \( d_i \). The simulation results of the coordinate transform between the WS and the RS by SMDM are shown in the last row of Table 2. From the table we can see that the SMDM produces identical simulation results. The three translation parameters \( a, b, \) and \( c \) and the three rotation parameters \( \alpha, \beta, \) and \( \gamma \) are all convergent to their relative actual value. The iterative error of the translation parameters is of the same order as the assumed error, while there is almost no iterative error of the rotation parameters.

| Table 2 Simulation results of the coordinate transform between the WS and the RS. |
|-------------------------------|-----------|-----------|-----------|-----------|-----------|
| Actual value                  | a (mm)    | b (mm)    | c (mm)    | \( \alpha \) (deg) | \( \beta \) (deg) | \( \gamma \) (deg) |
| Computed value \(^*\) 1 (if with 1-\( \mu \text{m} \) error) | 0.0000    | 0.0000    | 500.0000  | 45.0000   | 45.0000   | 60.0000   |
| Computed value \(^*\) 2 (if with 1-mm error)       | 0.0004    | 0.0005    | 500.0004  | 45.0000   | 45.0000   | 60.0000   |

\(^*\)Average value with the standard deviation \( \sigma \) of 0.11 \( \mu \text{m} \) for \( (a,b,c) \) and 0.00008 deg for \( (\alpha,\beta,\gamma) \); the successive calculation was repeated 100 times; \( \sigma = [(\nu^2)/(n-1)]^{1/2} \), where \( \nu \) is the residual value and \( n \) is the successive times.
Table 2. We can also see that the computed values $(a, b, c, \alpha, \beta, \gamma)$ are well convergent to their actual values. The iterative errors of $a$, $b$, and $c$ are of the order of the assumed error, while the iterative errors of $\alpha$, $\beta$, and $\gamma$ are less than 0.1 deg.

This proves that by proper arrangement of the calibration procedures, the SMDM algorithm is very stable for the self-calibration of the coordinate transform between the WS and the RS. And the iterative accuracy of the preceding method is of the same order of the measuring accuracy of interferometer.

$$
\begin{bmatrix}
R & T \\
O & 1
\end{bmatrix} = \begin{bmatrix}
\cos \gamma \sin \beta & \cos \gamma \sin \beta \sin \alpha - \sin \gamma \cos \alpha & \cos \gamma \sin \beta \cos \alpha + \sin \gamma \sin \alpha \\
\sin \gamma \cos \beta & \sin \gamma \sin \beta \sin \alpha + \cos \gamma \cos \alpha & \sin \gamma \sin \beta \cos \alpha - \cos \gamma \sin \alpha \\
-\sin \beta & \cos \beta \sin \alpha & 0
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c
\end{bmatrix} = \begin{bmatrix}
u_x & u_y & w_x & x_A \\
u_y & u_y & w_y & y_A \\
u_z & u_z & w_z & z_A
\end{bmatrix},
$$

(18)

where $u$, $v$, and $w$ are fixed with the target, so they are changed during measurement. The following equations give the calculation method:

$$
u = \frac{AB}{|AB|} = \frac{(x_B - x_A, y_B - y_A, z_B - z_A)^T}{[(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2]^{1/2}},$$

(19)

$$w = \frac{AB \times AC}{|AB \times AC|},$$

(20)

$$v = w \times u.$$  

(21)

From Eq. (18), two groups of solutions are obtained:

$$
\begin{align*}
sin \beta &= -u_z \\
cos \beta &= (1 - u_z^2)^{1/2} \\
sin \alpha &= \frac{u_z}{(1 - u_z^2)^{1/2}} \\
cos \alpha &= \frac{w_z}{(1 - u_z^2)^{1/2}} \\
sin \gamma &= \frac{u_y}{(1 - u_z^2)^{1/2}} \\
cos \gamma &= \frac{u_x}{(1 - u_z^2)^{1/2}}
\end{align*}
$$

$$
\begin{align*}
sin \beta &= -u_z \\
cos \beta &= (1 - u_z^2)^{1/2} \\
sin \alpha &= \frac{-u_z}{(1 - u_z^2)^{1/2}} \\
cos \alpha &= \frac{-w_z}{(1 - u_z^2)^{1/2}} \\
sin \gamma &= \frac{-u_y}{(1 - u_z^2)^{1/2}} \\
cos \gamma &= \frac{-u_x}{(1 - u_z^2)^{1/2}}
\end{align*}
$$

(22)

Which group is selected is a matter of practice. After calculation of the six parameters $(a, b, c, \alpha, \beta, \gamma)$ in each location of the object, the position and attitude of the target under the WS can be measured in real time.

### 3.1.5 Calculation of the attitude of the moving target under the WS

Sometimes, the characterization of the attitude relative to the initial position and attitude is more intuitive. So an IS is defined as the initial state of the TS. The homogeneous transform matrix from WS to IS, $T_{W-IS}$, can be calculated by means of the following equations:

$$
\begin{bmatrix}
\hat{x}_{1f} & \hat{x}_{2f} & \hat{x}_{3f} & \hat{x}_{4f}
\end{bmatrix} = T_{W-IS}[\hat{x}_{1W}, \hat{x}_{2W}, \hat{x}_{3W}, \hat{x}_{4W}],
$$

(23)

where $\hat{x}$ is the homogeneous coordinate vector in which the first three elements are the coordinates of $r$ and the fourth element is a constant 1:

$$
\begin{align*}
r_{1W} &= OA|_W \\
r_{1I} &= OA|_I = (0, 0, 0)^T \\
r_{2W} &= OB|_W \\
r_{2I} &= OB|_I = (x_B, 0, 0)^T \\
r_{3W} &= OC|_W \\
r_{3I} &= OC|_I = (x_C, y_C, 0)^T \\
r_{4W} &= (r_{2W} - r_{1W}) \times (r_{3W} - r_{1W}) + r_{1W} \\
r_{4I} &= (r_{2I} - r_{1I}) \times (r_{3I} - r_{1I}) + r_{1I}
\end{align*}
$$

(24)

Thus,

$$
T_{W-IS} = [\hat{x}_{1f}, \hat{x}_{2f}, \hat{x}_{3f}, \hat{x}_{4f}] [\hat{x}_{1W}, \hat{x}_{2W}, \hat{x}_{3W}, \hat{x}_{4W}]^{-1}.
$$

(25)

After knowing $T_{W-IS}$ in the initial state, the coordinates of targets $(x', y', z')^T$ under the IS can be calculated from the coordinates $(x, y, z)^T$ under the WS during measurement:

$$
[x', y', z', 1]^T = T_{W-IS} [x, y, z, 1]^T.
$$

(26)

Using the coordinates of targets under the IS, the six parameters of the target under the IS, $(x_A', y_A', z_A', \alpha', \beta', \gamma')$, can be obtained from Eqs. (19)–(22), which represents the position and attitude of the target relative to the initial state of target. It is obvious that the initial parameters of the target are $(0, 0, 0, 0)$ deg, $(0, 0, 0)$ deg.

From the preceding theoretical deduction, we can see that after the calibration of the initial distance and the self-
calibration of the coordinate transform between the WS and the RS, the attitude of the moving target under both the WS and the IS can be determined by the coordinates measured by each tracking station. As only linear computation is involved to deduce the attitude, so there is no simulation performed here.

3.2 DOM System

3.2.1 Algorithm

For one subtarget, the initial geometric parameters of the system, including the origins of the stations and the initial distances from the subtarget to the origins of the stations, can be self-calibrated by means of the redundancy using four tracking stations. For example, tracking stations 0, 1, 2, and 3 track subtarget A, as shown in Fig. 7, where \( A_j \) and \( A_{j+1} \) represent the \( j \)’th and \( (j+1) \)’th positions of the subtarget A. The geometric relation between the subtarget and the tracking stations is shown in Fig. 8, where \( O_i \) is the coordinate origin of the \( i \)’th station, which can be expressed as vector \( r_{ij} \); \( A_j \) is the \( j \)’th position of the subtarget, which is expressed by vector \( r_j \); \( l_{0j} \) represents the initial distance from the origin of the \( i \)’th station to the subtarget; and \( \Delta l_{ij} \) is the distance variation from the origin of the \( i \)’th station to the \( j \)’th position of the subtarget. The following formula can be obtained:

\[
(r_j - r_{si}) \cdot (r_j - r_{si}) = (l_{0j} + \Delta l_{ij})^2 \quad j = 1, 2, \ldots, N; \quad i = 0, 1, 2, \ldots, m - 1; \quad m = 4.
\]

The preceding formula includes \( 4N \) equations, where \( \Delta l_{ij} \) are the known parameters, the number of which is \( 4N \). The coordinates of the subtarget and the stations, and the initial distances are the unknowns, the number of which is \( 16 + 3N \) (= \( 3N + 3m + m \)). If the sampling number \( N \geq 16 \), all the unknowns in Eqs. (27) can be determined uniquely. Thus the initial distances and the origins of the coordinate systems can be self-calibrated.

When the DOM system is used to measure the attitude of the moving target, the following method to self-calculate the initial geometric parameters of the system is proposed. The tracking system is shown in Fig. 3 and all the coordinates of the stations and the subtargets are under the WS, thus the coordinate transform between the WS and the RS is unnecessary.

First, the coordinates of the six stations are self-calibrated while six stations track subtarget A simultaneously. The equations, which are similar to the Eqs. (27), can be adopted to determine the unknown parameters of the six stations when \( N \geq 8 \), where \( m = 6 \), the number of equations is \( 6N \), and all the geometric parameters of the system are unknown whose number is \( 3N + 24 \).

Second, the initial distance can be self-calibrated while four of six stations track subtargets \( A, B, \) and \( C \) respectively. In step 1, stations 0, 1, and 2 can be self-calibrated when \( N \geq 4 \). In step 2, stations 3 and 4 and any two of other stations track subtarget \( B \). The initial distances from the stations 3 and 4 to subtarget \( B \) can be determined by means of the same method.

In step 3, station 5 and any three of other stations track subtarget \( C \). The initial distance from the station 5 to subtarget \( C \) can be determined. In this method, we assume that the target can be returned to its initial status at the beginning of each step, so the initial distance it is unnecessary to self-calibrate again when the station renews to track the subtarget after tracking another subtarget following self-calibration of the initial distance.

Finally, the coordinate measurement of the combination target system can be completed in real time after self-calibration. As shown in Sec. 2.2, the coordinates of subtarget A are measured using stations 0, 1, and 2; the coordinates of subtarget \( B \) are measured using stations 3 and 4; and the coordinates of subtarget \( C \) are measured using tracking station 5. Simultaneously, the attitude of the combination target system under the WS and the IS can also be calculated in real time by the method described in Secs. 3.1.4 and 3.1.5.

3.2.2 Simulation

In Fig. 8, let the coordinates of the four stations be \( (x_{S1}, y_{S1}, z_{S1}), (x_{S2}, y_{S2}, z_{S2}), (x_{S3}, y_{S3}, z_{S3}), \) and \( (x_{S4}, y_{S4}, z_{S4}) \), and let the initial lengths from the stations to the target be \( l_{01}, l_{02}, l_{03}, \) and \( l_{04} \). For simplicity, let the coordinates of the four stations \( O_1 \) at the origin of coordinate system \((0,0,0)\), \( O_2 \) at the \( x \) axis \((1000,0,0)\), \( O_3 \) on the \( x-y \) plane \((1000,1000,0)\), and \( O_4 \) at \((1000,1000,500)\), respectively. The unit of a coordinate is millimeter. The target moves from point \((500,500,500)\) to point \((800,500,500)\), and the locus of the moving path is a line parallel to the \( x \) axis and the number of the sampling points is \( 100 \). The Levenberg-Marquardt method\(^{17}\) was adopted to solve the nonlinear Eqs. (27) and the iterative values of the 10 unknowns \((l_{01}, l_{02}, l_{03}, l_{04}, x_{S2}, x_{S3}, y_{S3}, x_{S4}, y_{S4}, z_{S4})\) of the system were achieved and are shown in Table 3. The
deviations of the initial estimated values are 500 μm from their corresponding real values. The computed value 1 was obtained by assuming that there is 0 to 1 μm random measurement error of the interferometer to measure the displacement of the target; while the computed value 2 was obtained by assuming the measurement error of the interferometer to be 10 μm.

From Table 3, we can see that even though the estimated initial values deviate about 500 μm from their real values, the iterative values of the unknowns can still be well convergent to their real values. The iterative accuracy is of the same order as that of the supposed accuracy of the laser interferometer. It can be said that the self-calibration algorithm using Levenberg-Marquardt method is very stable and is well suitable to calibrate the initial geometric parameters of the DOM system.

4 Summary and Discussion

We have demonstrated through modeling and calibration of the DAM and DOM systems that by (1) using three stations and three targets for DAM system and (2) using six stations and three targets for DOM system, both systems are able to perform the full attitude measurement of the moving object. By analytical deduction and numerical simulation, we demonstrated the following underlying principles to measure the full attitude of moving object by multi-station laser tracking systems:

1. The target should be a combination of at least three subtargets. The combination target system is simple and easy to use in industry. For DAM system, at least three tracking stations are required, while at least six tracking stations are required for DOM system.

2. The rotation and translation of different tracking stations can be self-calibrated. The self-calibration algorithm is the key factor for the attitude measurement by the proposed systems, which converges rapidly and has a satisfactory accuracy performance under noise measurement. The proposed self-calibration approach can provide a tool for rapid and autonomous calibration of laser tracking systems.

3. The self-calibration algorithm for the DAM system can be solved by the SMDM, while for the DOM system, the nonlinear equations to self-calibrate the system parameters can be solved by the Levenberg-Marquardt method.

4. After the measurement of the coordinates, the full attitude of the moving object can be expressed both under the WS and the IS by linear computation.

Many open research issues remain along the direction of the proposed approaches for the attitude measurement by laser tracking systems. The first issue is how to decide the optimum arrangement of the tracking stations. For instance, Takatsuji et al.18 suggested that four laser trackers should not locate at the same plane, otherwise the self-calibration algorithm will be ill-conditioned and the estimated system parameters are incorrect. This is why we did not let the four stations locate in the same plane in the simulation in Sec. 3.2.2 and we obtained satisfactory results. But it is still necessary to further clarify how the arrangement of the different number of tracking stations influences the accuracy enhancement of the whole system.

Another equally important issue is to decide an optimum moving path of the target for self-calibration. For instance, in Sec. 3.1.3, let the target move along a line parallel to x axis, from point (100,100,100) to point (700,100,100), while the other conditions be the same as those for Table 2, however, the simulation results are totally different from those in Table 2, as shown in Table 4. It shows that the matrix H in Eq. (17) cannot be singularly decomposed and the algorithm cannot converge to the correct values when the trajectory of the target and the two stations are located in the same plane. Also the moving path of the target may influence the arrangement of the tracking stations, such as

Table 4 Simulation results of the coordinate transform between the WS and the RS when target moves along a line parallel to x axis.

<table>
<thead>
<tr>
<th></th>
<th>a (mm)</th>
<th>b (mm)</th>
<th>c (mm)</th>
<th>α (deg)</th>
<th>β (deg)</th>
<th>γ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual value</td>
<td>0.0000</td>
<td>0.0000</td>
<td>500.0000</td>
<td>45.0000</td>
<td>45.0000</td>
<td>60.0000</td>
</tr>
<tr>
<td>Computed value</td>
<td>-43.5414</td>
<td>185.3409</td>
<td>639.7392</td>
<td>45.0000</td>
<td>24.3421</td>
<td>78.2514</td>
</tr>
</tbody>
</table>

in some ranges the stations cannot track the target or make the self-calibration algorithm close to be ill-conditioned. Therefore, the moving path of the target should be properly considered during the self-calibration of the coordinate transform between the WS and the RS.

The other important issue is how to decide the optimum number of sampling points and the optimum preestimated values of the initial parameters. The increase of the number of sampling points may increase the accuracy of self-calibration algorithm in some sense. Table 5 shows the simulation results when the number of sampling points is 15. The measuring accuracy of interferometer is supposed to be 1 μm. The other conditions are the same as those for Table 3. Compared the simulation results of Tables 5 and 3, it can be seen that the number of sampling points does affect the iterative accuracy. But the increase of the number of sampling points will take more computation time, which may limit the dynamic tracking and measuring performance of the whole system. Meanwhile according to the simulation results, the requirement of the preestimated values of the initial parameters of the DOM system is not so high, even though there is a 0.5-mm deviation the algorithm is able to converge to the real value, but the different preestimated value can influence the convergent time. Thus, in the design of the practical system, the computation time should be considered so that the system has the performance of both fast response and high accuracy.

Although the feasibility of the full attitude measurement of a moving target by DAM and DOM systems has been demonstrated theoretically and the mathematical model for the calculation of the attitude has been built in this paper, the proposed approaches also need to be tested extensively in a variety of real systems. The full potential of the approaches remain to be explored. We think the following guidelines can be followed to further simplify the calibration algorithm of the initial parameters and the measurement procedures for practical applications:

1. The complication and time-consuming of algorithms for the DAM and DOM systems to measure the full attitude of the moving target may be settled by the introduction of optimal control algorithm, the simplification of mathematical model of system and the usage of powerful computer.
2. The usage of “cat’s eye” retroreflectors will increase the measuring accuracy and reduce the restriction of the arrangement of laser trackers. This kind of retroreflector can be made with the materials that have the refractive index \( n = 2.0 \).
3. The development of absolute distance laser measurement technique\(^8\) may lead to more simple structure of laser tracking systems, as absolute interferometers will enable the measurement procedures to be easier and it is no need for calibration of the initial distances before measurement. Also, the measurement procedures can be conducted continually even though the laser beam is blocked instantly.

It is reasonable to believe that the attitude measurement method mentioned above would be benefit to industrial, manufacturing, novel metrology in dynamic condition, and other moving parts in 3-D space. We will continue to resolve the preceding problems and perform the practical experiments of the attitude measurement of moving target.

References

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